Classifying facilitators' multi-layered positions in teachers' collaborative work: Experiences from a lesson study project in the UK

Yukiko Asami-Johansson[1], Jacob Bahn[2], Koji Otaki[3] and Ruth Trundley[4]

[1]University of Gävle, Faculty of Engineering and Sustainable Development, Gävle, Sweden; yuoasn@hig.se

[2] Independent researcher and developer of mathematics teaching, Denmark, [3]Hokkaido University of Education, Japan, [4] Loughborough University, United Kingdom

This paper explores how facilitators' positions in lesson study can be distinguished and classified through the lens of the Anthropological Theory of the Didactic. Using the concepts of paradidactic system and paradidactic infrastructure, we develop a framework for analysing facilitation at different levels of collaborative teacher learning. A lesson study project in the United Kingdom is used as an illustrative case to exemplify the proposed conceptual distinctions. The analysis shows that facilitators may occupy multiple positions simultaneously within nested didactic and paradidactic systems, with some supporting teachers' inquiry into lessons and others contributing to the development of the infrastructures that sustain such inquiry. By distinguishing between first- and second-order facilitative functions, the paper offers theoretical tools for understanding how lesson study is established, maintained, and developed across educational contexts.

Keywords: Lesson study, facilitation, anthropological theory of the didactic, paradidactic system, collaborative settings in mathematics teacher education.

Introduction

Recent decades have witnessed growing interest in collaborative settings in mathematics teacher education (CSMTE) as a means of supporting teachers' professional learning. In particular, lesson study, originating in Japan, has attracted considerable international attention, and its principles and working methods have been adapted in many educational contexts to support the development of teachers' knowledge of mathematics teaching and learning (e.g., Lewis et al., 2009; Takahashi & McDougal, 2016). As lesson study becomes increasingly established beyond Japan, researchers have also sought theoretical tools for analysing the conditions that enable and sustain such collaborative forms of professional development (e.g. Winsløw et al., 2018). Within the anthropological theory of the didactic (ATD), teachers' collaborative work in lesson study has been conceptualised as a form of paradidactic action or *paradidactic systems* (Winsløw, 2012; Miyakawa & Winsløw, 2013. We will give more detailed explanation later). From this perspective, lesson study is not merely a professional development activity but a specific "didactic" system in which teachers collectively study and investigate teaching and learning. Any set of the institutional conditions that is developed to support such paradidactic activities have been described as a *paradidactic infrastructure* (Winsløw, 2012), providing an analytical framework for examining both the conditions and constraints under which the paradidactic system of collaborative teacher learning functions or does not function.

A key component of lesson study is the work of *facilitators* who support and organise teachers' collective inquiry. Depending on the context, facilitators may be researchers, teacher educators,

experienced teachers, school leaders, or other individuals involved in the organisation and development of lesson study. Their contributions extend beyond supporting individual lesson study cycles and may include establishing and maintaining the infrastructures that make such cycles possible. In this paper, we examine how facilitators' roles can be distinguished and classified according to their *positions* (Chevallard, 2019) within different "levels" within paradidactic systems. Using a lesson study project in the United Kingdom as an illustrative case, we develop and exemplify this conceptualisation. The paper is primarily theoretical in nature, aiming to refine the conceptual tools available for analysing different types of facilitation in lesson study and its paradidactic systems for teachers' collaborative work. The UK project is therefore not analysed as an empirical case in its own right; rather, it serves as a vehicle for illustrating and elaborating the proposed framework. Accordingly, contextual and methodological details are included only insofar as they support the theoretical argument.

We argue that researchers investigating the mechanisms and conditions that sustain lesson study need to consider the complex, inter-connected and multi-level nature of these systems. Such an analysis is particularly relevant in the current international educational context, where lesson study has become a global phenomenon and where changing teachers' working conditions, together with a growing emphasis on inquiry-oriented professional development, require us to reconsider how lesson study is organised, supported, and further developed. We begin by outlining the ATD concepts of paradidactic system and paradidactic infrastructure before using the UK lesson study project to illustrate how facilitators may occupy different positions within first- and second-order paradidactic systems.

Theoretical framework: Paradidactic systems and infrastructures in ATD

The Anthropological Theory of the Didactic (ATD) describes teaching and learning through the notion of a *didactic system*, represented as $S(\hat{x}, \hat{y}; \heartsuit)$ (Chevallard, 2020), where ♥ denotes the object of study (a didactic stake), $\hat{x}$ the "student instance" (i.e., an individual student, the collective of students, or the *student position*), and $\hat{y}$ the "teacher instance" (i.e., an individual teacher, the collective of teachers, or the *teacher position*)[1]. In a mathematics classroom, the didactic stake is typically a mathematical topic, the persons who occupy the student position constitute the student instance, and the teacher supports students' study of that topic. The operation of such a system corresponds to what is commonly understood as a teaching-learning situation.

ATD further distinguishes *paradidactic systems*, in which the object of study is itself a didactic system. Formally, a paradidactic system may be represented as $S(\hat{x}', \hat{y}'; S(\hat{x}, \hat{y}; \heartsuit))$ (cf. Otaki, 2024). Here, participants do not directly study mathematics; rather, they investigate the teaching and learning processes that occur (or not) within a didactic system at stake. Activities such as lesson planning, lesson observation, post-lesson discussions within lesson study can therefore be understood as paradigmatic examples of the functioning of paradidactic systems. In these settings, teachers collectively examine and reflect on teaching practices with the aim of improving instruction. This distinction between didactic systems and paradidactic systems leads us to identify different levels, or orders of paradidactic systems themselves (Otaki, 2026). A *first-order paradidactic system*, denoted

[1] In this paper, we mainly use the symbols "$\hat{x}$" and "$\hat{y}$" (and their "derivative" symbols) for indicating the student position and the teacher position, respectively.

as $S(\hat{x}', \hat{y}'; S(\hat{x}, \hat{y}; \heartsuit))$, arises when teachers collaboratively study a didactic system, such as a lesson. In first-order systems of lesson study teachers observe, analyse, and discuss classroom events, making the lesson itself the object of collective inquiry. By contrast, in a *second-order paradidactic system*, the first-order paradidactic system itself becomes the object of study. Hence, a new "meta-level" system emerges, which we can denote as: $S(\hat{x}'', \hat{y}''; S(\hat{x}', \hat{y}'; S(\hat{x}, \hat{y}; \heartsuit)))$. At this level, participants are no longer primarily investigating a particular lesson but rather the organisation, functioning, and development of *lesson study itself*. Researchers, organisers, teacher educators, and other facilitators may therefore study how lesson study supports teachers' learning, how it is built and sustained over time, and which conditions enable its successful implementation.

To account for such conditions, ATD introduces the notion of paradidactic infrastructure. Paradidactic infrastructures comprise the organisational, institutional, and intellectual resources that support the operation of paradidactic systems. Examples include established routines for lesson observation and discussion, professional networks, facilitation practices, research knowledge, and organisational arrangements that enable teachers to participate in collaborative inquiry. From this perspective, *facilitators* play an important role not only *within* but also *around* lesson study in establishing and maintaining the infrastructures that make lesson study possible.

As previously stated, we use these concepts to categorise the different levels of facilitator involvement in a lesson study project in the United Kingdom. More specifically, we explore how these facilitators occupy distinct positions within first- and second-order paradidactic systems and how these positions relate to the development of the paradidactic system and the expansion of the paradidactic infrastructure that supports teachers in their lesson study.

The context: A lesson study project in the UK

Collaborative Lesson Research UK (CLR-UK) is a charitable organisation that promotes and supports lesson study and collaborative lesson research across the United Kingdom. Its work focuses on creating opportunities for teachers, school leaders, researchers, and other educational professionals to engage in and sustain lesson study as a form of professional learning. Through partnerships with schools, universities, and educational organisations, CLR-UK seeks to develop lesson study practices suited to the UK context while also fostering connections between research, classroom practice, and international lesson study communities, including Japan.

The event discussed in this paper emerged from a longer-term collaboration between CLR-UK and teachers from Tsukuba University Elementary School in Japan. Through a series of exchange visits, the format of these events evolved to combine opportunities to learn about Japanese mathematics education with the observation and discussion of a research lesson. The specific lesson analysed here was selected following a CLR-UK study visit to Japan in February 2026, involving educational professionals from several European countries. During that visit, participants observed a Grade 4 lesson at the Tsukuba open research event and subsequently saw the same lesson taught in a different school in Osaka. Recognising the potential of the lesson for supporting professional inquiry, CLR-UK invited Mr. Ono from Tsukuba University Elementary School to teach the lesson in a primary school near London.

From the perspective of ATD, CLR-UK can be understood as occupying the position of $\hat{y}''$ within the second-order paradidactic system $S(\hat{x}'', \hat{y}''; S(\hat{x}', \hat{y}'; S(\hat{x}, \hat{y}; \heartsuit)))$. Rather than focusing on the improvement of a particular lesson, the organisation works to establish the conditions that enable collaborative lesson research to occur and develop. Its activities therefore contribute to the creation and maintenance of a paradidactic infrastructure through which lesson study can be sustained, adapted, and expanded within the UK context.

The case of research lesson and post-lesson discussions in the UK

In this section, we describe the open research lesson carried out at Three Bridges Primary School located outside London. Before the pupils would enter the classroom, the Tsukuba teacher Ono, began with a brief explanation of his intentions and the focus of the lesson for the UK participants (teachers from Three Bridges and other schools and teacher educators from across the UK). He explained that the pupils would investigate the relationship between two geometric figures.

The lesson

The lesson was conducted through consecutive interpretation. An interpreter (the second author) translated Mr. Ono's Japanese instructions, questions, and comments into English for the pupils and translated the pupils' responses from English into Japanese for Mr. Ono. In a patient and deliberate manner, Mr. Ono guided students through observing, discussing, and determining the total number of sticks used to construct squares with side lengths of 1, 2, and 3 sticks. Students explored different solution strategies, including counting and multiplication, expressing the total number of sticks as the number of sticks on one side multiplied by the four sides of the square. The activity concluded with students explaining how to determine the number of sticks in a square with 10 side lengths.

The lesson then shifted to staircase-shaped figures. Beginning with a one-step staircase and progressing to larger examples, students considered both the number of sticks along the side, i.e. the base, and the total number of sticks in each figure's perimeter. After introducing a three-step staircase, Mr. Ono asked whether students could determine the total number of sticks without counting. One student proposed the calculation 3×4, prompting a discussion about whether this multiplicative structure can be seen directly in the staircase as it can in a square. By comparing staircases and squares with the same base lengths and total numbers of sticks, Mr. Ono guided students to recognize their structural similarity. Students demonstrated different ways of transforming a staircase into a square, most notably by sliding sticks from the perimeter of the staircase to the perimeter of the square. Through this process, the multiplicative structure becomes visible.

In the end, students determined the number of sticks in a staircase with 10 steps and explained how the calculation 10×4 can be seen in the figure. The lesson concluded with the teacher asking about a staircase with 100 steps, to which the students confidently responded, "400." Mr. Ono summarised that transforming the staircase into a square makes the multiplication underlying the pattern visible.

The post lesson discussions

After the pupils had left the classroom, a post-lesson discussion panel was convened to stimulate and guide the discussion among participants. The panel consisted of Mr. Ono, three colleagues from Tsukuba University Elementary School, and one of the facilitators from CLR-UK. Communication

between the Japanese teachers and the UK participants was facilitated by an interpreter (the first author), who translated the UK participants' comments into Japanese and the Japanese teachers' comments into English.

Mr. Ono began by reflecting on the lesson. Among other points, he noted that time constraints had made it difficult to fully develop the intended comparison between the two figures. His goal had been for pupils to discover that the staircase-shaped figure could be transformed into a square and that the same multiplicative relationship could therefore be used to determine the number of sticks. He also commented on the challenge of teaching in English, explaining that his inability to fully understand pupils' comments as they arose limited his capacity to respond spontaneously to classroom situations in the way he would normally do in Japan.

Mr. Ono encouraged the participants to engage actively in discussing both the strengths and the limitations of the lesson. He expressed his hope that the lesson would serve as a *reference point and research resource* for teachers developing their own problem-solving lessons. In these comments, he occupied two different positions simultaneously. First, he acted as an $\hat{x}$' within the first-order paradidactic system $S(\hat{x}', \hat{y}', S(\hat{x}, \hat{y}; \heartsuit))$ by reflecting upon his own didactic practice. Second, he acted as even a $\hat{y}''$ within the second-order system $S(\hat{x}'', \hat{y}''; S(\hat{x}', \hat{y}'; S(\hat{x}, \hat{y}; \heartsuit)))$, by supporting the participants' study of the lesson discussion itself and thereby contributing to the development of the paradidactic system. Mr. Ono also responded to several questions concerning the rationale behind his instructional decisions, such as: *"Why did you choose these two figures as the starting point for pupils' investigation?"* In doing so, in the first-order system, he simultaneously occupied the position of $\hat{x}$', critically examining his own actions, and $\hat{y}$', acting as a knowledgeable other who supported the participants' inquiry into the didactic system $S(\hat{x}, \hat{y}; \heartsuit)$.

The facilitator from CLR-UK also occupied a position of $\hat{y}$' in the first-order system by drawing attention to Ono's focus on mathematical structure, relating it to a characteristic feature of Japanese mathematics education in which pupils are encouraged to identify invariants and generalise across apparently different situations.

The other Tsukuba teachers began by expressing considerable respect for Mr. Ono's lesson design and pedagogical expertise. For example, one teacher observed:

> "*When a pupil was modifying the figure, Mr. Ono was not looking at the whiteboard. Instead, he was watching the class because he already knew how the figure would be modified. What interested him was how the other pupils would respond to their classmate's action*."

The discussion then moved beyond appreciation to consideration of alternative instructional approaches. Another Tsukuba teacher suggested:

> "*I interpreted the lesson as an introduction to the concept of functions. If that was the intention, I would have emphasised more explicitly how changing one quantity affects another. I would also have highlighted the figures and numerical relationships to make clear that the quantities remained equivalent*."

Mr. Ono responded by clarifying his own intentions: "*My aim was to have pupils recognise that the staircase figure could be transformed exactly into a square. Therefore, I did not focus on that aspect*

At this point, the Tsukuba teachers can be understood as occupying the position of $\hat{x}$', collectively exploring alternative approaches to teaching through discussion within the first-order system. At the

same time, however, they also functioned as $\hat{y}$' by *demonstrating* what a productive post-lesson discussion *might look like*. Through their questions, analyses, and alternative proposals, they demonstrated how participants can contribute to the refinement of a lesson by making teachers' intentions explicit and open to critical examination. Moreover, these contributions also functioned at the level of the $\hat{y}$'' within the second order system because the UK participants were not only learning about the lesson itself but also observing how experienced lesson study practitioners conduct post-lesson discussions. This was reflected in the school principal's remarks:

> "*Mr. Ono's lesson itself was impressive, but listening to and watching your interaction was perhaps even more valuable. The professionalism shown throughout the discussion was like magic. It made me think about how we could develop this kind of discussion ourselves. People confidently propose alternative approaches and engage critically with one another's ideas*."

Mr. Ono's response further highlighted this second-order facilitative role:

> "*The purpose of lesson study is to improve teaching, not simply to judge whether a lesson was good or bad. A lesson is only one possible model; what matters is how it might be improved through alternative approaches. Although everyone has been respectful of my lesson today, we need more discussion about how we would do things differently. Such discussions improve lessons and, ultimately, children's mathematical abilities. Constructive criticism is therefore essential."*

In this statement, Mr. Ono clearly acted as $\hat{y}$'' of the second-order system: He was explicitly communicating to participants what constitutes productive lesson study practice.

Another participant who can be interpreted as occupying the position of the $\hat{y}$'' in the second-order paradidactic system (as well as $\hat{y}$ in the didactic system) was the lesson *interpreter*. During the research lesson, the interpreter's responsibility was to translate Mr. Ono's instructions, questions, and comments as accurately as possible, without adding personal explanations or interpretations. This was particularly important because additional explanations could unintentionally reveal pedagogical intentions that Mr. Ono had deliberately left implicit for the pupils to discover through their own inquiry (or infuse some, not intended by Mr. Ono). In this way, the interpreter supported the functioning of the didactic system $S(\hat{x}, \hat{y}; \heartsuit)$ while simultaneously contributing to the conditions that enabled the lesson study event to operate effectively across linguistic and cultural boundaries.

Discussions and conclusion

The analysis highlights that facilitation in lesson study cannot be understood as a single, uniform role. Rather, facilitators occupy different positions within nested didactic and paradidactic systems, and their functions vary according to the object of study at each level.

Within the research lesson itself, participants such as Mr. Ono acted as teacher position $\hat{y}$ in the didactic system $S(\hat{x}, \hat{y}; \heartsuit)$, supporting pupils' study of mathematics. During the lesson and subsequent post-lesson discussion, Mr. Ono participated in the first-order paradidactic system $S(\hat{x}', \hat{y}'; S(\hat{x}, \hat{y}; \heartsuit))$; as a $\hat{x}$', he was a learner, whose own lesson served as the object of reflection and inquiry, and as a $\hat{y}$', an experienced lesson study practitioner. He contributed to supporting other participants' inquiry into the lesson, thereby assuming a facilitative function within the same paradidactic system. Mr. Ono took an act even as a $\hat{y}$'' in the second-order paradidactic system $S(\hat{x}'', \hat{y}''; S(\hat{x}', \hat{y}'; S(\hat{x}, \hat{y}; \heartsuit)))$, by

supporting the participants' study of the lesson discussion itself. This example illustrates that a single individual may occupy multiple positions in the first- and second-order systems (also across didactic and paradidactic systems) and that these positions are defined by their relation to the object of study rather than by formal professional status alone.

The analysis also reveals a distinction between facilitators operating at the first and second paradidactic orders. The contributions of teachers and knowledgeable others during lesson observation and post-lesson discussion primarily supported inquiry into the didactic system. Their role was therefore linked to the first-order paradidactic system. In contrast, participants, such as the principal, were not only interested in the research lesson itself but also in how the Japanese and UK participants collectively analysed and discussed the lesson. His observation that the most interesting aspect was the interaction among the Tsukuba teachers exemplifies a second-order perspective, in which the paradidactic system itself becomes the object of the study. Similarly, CLR-UK itself can be interpreted as occupying a $\hat{y}''$ position within a second-order paradidactic system. While individual organisers/facilitators contributed to the event through particular facilitative actions, CLR-UK functioned at an organisational level by creating the conditions that made the lesson study possible. Its activities included establishing partnerships with Japanese educators, organising international exchanges, identifying and inviting participants, and providing structures through which teachers could engage in collaborative inquiry. The event analysed in this paper can therefore be understood not simply as the result of individual facilitation but as an expression of a broader paradidactic infrastructure developed and maintained by CLR-UK. The organisation played a key role in creating and sustaining the institutional, organisational, and intellectual conditions that enabled first-order paradidactic activity to occur.

This distinction between first-order and second-order facilitation has implications for research on collaborative settings in mathematics teacher education. Studies of lesson study often focus on interactions within a lesson study cycle, yet our analysis suggests that these interactions depend upon broader infrastructures that are frequently invisible. The case of CLR-UK illustrates that second-order facilitation may be exercised not only by individuals but also by organisations that support and coordinate collaborative inquiry across contexts and over time.

Facilitators operating at this level contribute not only to particular lesson study events but also to the long-term development of the systems that support them. We therefore argue that analysing lesson study through nested paradidactic systems of successive orders provides a useful framework for understanding the variety of facilitative roles involved in its development. Such a framework enables researchers to examine a paradidactic system from a position outside that system itself. In other words, each higher-order paradidactic system offers an external perspective on the functioning of the level below, making visible features, norms, and conditions that may remain difficult for participants to perceive from within. For example, researchers positioned outside a second-order paradidactic system may investigate how lesson study is organised, sustained, and transformed across contexts, thereby bringing into view aspects of the paradidactic infrastructure that would otherwise remain largely implicit. This perspective helps distinguish between those who support teachers' inquiry into lessons and those who contribute to the institutional and organisational conditions that make such inquiry possible. Such distinctions are particularly important in contemporary educational systems, where collabo

rative professional learning increasingly relies on networks, organisations, and individuals working across institutional boundaries. We conclude that facilitators' roles in lesson study are best understood not as fixed professional identities but as *positions* within interconnected didactic and paradidactic systems. These positions may be occupied by individuals, groups, or organisations, and may shift as the object of study changes across levels. Recognising these layered positions enables a richer analysis of how lesson study is initiated, sustained, and transformed over time, as well as how the infrastructures supporting it are developed and renewed.

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